# Line equation

Any line can be described by an equation:

Where *m* is the gradient and *b* is the value of *y* when *x* = 0.

This is the “slope-intercept” form which often treats the line as a graph of a function. We can re-write this equation in a more even-handed way that treats the line as a geometrical object. The more general form (normal form) of the equation is:

If the coefficients of the normal form are multiplied by a nonzero constant *k* then the set of solutions is the same, in other words the equation still has the same solutions – it describes the same line:

## Finding the equation of a line using 2 points in the plane

For any points *P* and *Q* there is exactly one line *PQ* that connects them. If the coordinates of *P* and *Q* are known, then the coefficients *a*, *b* and *c* of the line equation can be discerned as follows.

**Example:** Suppose *P* = (1,2) and *Q* = (-2,5). Find the equation that describes the line *PQ*:

* Since *P* is on the line its coordinates must satisfy the equation: , or
* Similarly, since *Q* is also on this line its coordinates must satisfy the equation
* We can rewrite the first equation as . If we substitute this into the second equation, we can obtain
* Substituting that equation for *b* into the first line equation, we can obtain:
* Finally, we can write the line equation as .

# Plane equation

A plane in 3D space has the equation:

Similarly, to the line equation, if it is multiplied by a nonzero constant *k* then the plane of solutions is the same.

This can also be written as:

Where (*x0*, *y0*, *z0*) is any point on the plane. The vector <*a,b,c*> would be a vector that is normal (perpendicular to the plane). We can then derive the plane equation as follows:

1. Using a least 3 points on the plane (*P,Q,R*) we can derive two vectors that are planar with the plane.
2. We can take the cross product of those vectors, which will produce another vector which is normal to the plane. This gives us the values for *a*, *b* and *c*.
3. We can substitute in and of the three points *P*, *Q* or *R* into the equation in place of *x0*, *y0 and z0.* This gives us the final equation.

**Example:** find the equation of the plane that passes through the points P(1,0,2), Q(-1,1,2) and R(5,0,3)

1. Derive two vectors:
2. Find the cross-product (derivation not shown here):
3. Substituting into the empty plane equation (using point *P* in this example):